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INFLUENCE OF HEAT AND MASS TRANSFER ON THE DEVELOPMENT OF TWO-DIMENSIONAL SEPARATED FLOWS

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The effect of cooling and heating of a streamlined surface, free mass transfer, weak and strong isothermal injection, and suction on the development of supersonic turbulent separated flows is considered. The influence of the temperature factor and inefficiency ratio parameter on the gas-dynamic and geometric characteristics of separated turbulent flows is estimated.

1. Introduction. The problem of heat and mass transfer effect in turbulent separated flows is very important in the protection on structural elements from high-temperature flows, control of flow separation, and optimization of aerodynamic forms. So far, integral methods and experiments have played a leading role [1-6]. In [3] the effect of distributed injection on supersonic flow separation with different generators of shock waves is studied to check experimentally the concept of "free interaction" under conditions of porous injection. The effect of streamlined surface cooling on generation and development of supersonic turbulent separation was studied in [4, 6]. In turn, the means of computational hydroaerodynamics, developed at present and mutually supplementing the experimental approach, allow one to obtain detailed information on the structure and characteristics of viscous-inviscid separating interactions [2, 5, 7-14].

On the basis of the Beam–Worming method and a modification of improved accuracy of the Steger implicit factored scheme, which are realized within the same program package [9, 14], we conducted a comparative study of the effect of cooling and heating of a streamlined surface and weak distributed mass transfer and also strong concentrated injection on the gas-dynamic and geometric characteristics of separated turbulent flows.

2. Problem Formulation. It is assumed that the employment of the complete nonstationary Navier–Stokes equations is the most universal and plausible approach in numerical simulation of hydrogas-dynamic processes. To calculate flows in the presence of flow separation and heat and mass transfer we use the notation of initial equations in arbitrary curvilinear coordinates for dimensional variables:

$$\frac{\partial \hat{q}}{\partial t} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} = \frac{1}{\mathrm{Re}} \left(\frac{\partial \hat{T}}{\partial \xi} + \frac{\partial \hat{S}}{\partial \eta} \right), \tag{1}$$

where

$$\hat{q} = \frac{1}{J} \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (e + p)\, u \end{bmatrix}, \quad F = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ (e + p)\, v \end{bmatrix}, \quad \begin{aligned} \hat{E} &= (\xi_x E + \xi_y F)/J, \\ \hat{F} &= (\eta_x E + \eta_y F/J, \end{aligned}$$

$$T = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ S_x \end{bmatrix}, \quad S = \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ S_y \end{bmatrix}, \quad \begin{aligned} \hat{T} &= (\xi_x T + \xi_y S)/J, \\ \hat{S} &= (\eta_x T + \eta_y S)/J, \\ J &= \frac{D(x, y)}{D(\xi, \eta)} \end{aligned}$$

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$$\tau_{xx} = (\lambda + 2\mu)\, u_x + \lambda v_y, \quad S_x = u\tau_{xx} + v\tau_{xy} + \frac{k}{\Pr(\gamma - 1)} \frac{\partial a^2}{\partial x},$$

$$\tau_{xy} = \mu\,(u_x + v_y), \quad S_y = u\tau_{xy} + v\tau_{yy} + \frac{k}{\Pr(\gamma - 1)} \frac{\partial a^2}{\partial y},$$

$$\tau_{yy} = (\lambda + 2\mu)\, v_y + \lambda u_x, \quad p = (\gamma - 1)\left[e - \frac{\rho}{2}(u^2 + v^2)\right],$$

$$a^2 = \gamma \frac{p}{\rho}, \quad \mu = \mu_\infty\,(T/T_\infty)^{0.76}.$$

In the formulation of the boundary conditions on the inlet loop of the computation region the parameters of the oncoming undisturbed flow were fixed. An incident shock wave was generated by assignment of the parameters of the flow behind an oblique shock wave on the upper boundary of the computation region. On the outlet portion of the computation region, "soft" boundary conditions in the form of the absence of longitudinal gradients of gas-dynamic parameters were formulated. On the streamlined surface in the boundary layer the conditions of sticking and the heat and mass transfer parameters were assigned, and, moreover, a boundary layer approximation on the absence of a longitudinal pressure gradient in the viscous sublayer was used here. Injection of a concentrated sound jet (M = 1) was modeled by assignment of the jet parameters in the corresponding nodes of the computation grid.

3. **Numerical Method.** To construct a numerical algorithm for solving Eqs. (1), as in [7], the derivatives for the convective terms were approximated by central differences

$$\begin{aligned} \frac{\partial \hat{E}}{\partial \xi} &\approx \delta_\xi \hat{E} = \frac{\hat{E}_{j+1} - \hat{E}_{j-1}}{2\Delta\xi} = \left(1 + \frac{\Delta\xi^2}{6}\frac{\partial^2}{\partial\xi^2}\right)\frac{\partial \hat{E}}{\partial \xi} + O(\Delta\xi^4), \\ \frac{\partial \hat{F}}{\partial \eta} &\approx \delta_\eta \hat{F} = \frac{\hat{F}_{k+1} - \hat{F}_{k-1}}{2\Delta\eta} = \left(1 + \frac{\Delta\eta^2}{6}\frac{\partial^2}{\partial\eta^2}\right)\frac{\partial \hat{F}}{\partial \eta} + O(\Delta\eta^4) \end{aligned} \tag{2}$$

and for the viscous terms by finite-difference relations of the form

$$\frac{\partial}{\partial\xi}\left(\Phi \frac{\partial}{\partial\xi}\Psi\right) \approx \delta_\xi(\Phi\delta_\xi\Psi) = \frac{1}{2(\Delta\xi)^2}\left[(\Phi_{j+1} + \Phi_j)(\Psi_{j+1} - \Psi_j) - (\Phi_j + \Phi_{j-1})(\Psi_j - \Psi_{j-1})\right]_k,$$

$$\begin{aligned} \frac{\partial}{\partial\xi}\left(\Phi \frac{\partial}{\partial\eta}\Psi\right) \approx \delta_\xi(\Phi\delta_\eta\Psi) = \frac{1}{4\Delta\xi\Delta\eta}\Big[\Phi_{j+1,k}(\Psi_{j+1,k+1} - \Psi_{j+1,k-1}) - \\ - \Phi_{j-1,k}(\Psi_{j-1,k+1} - \Psi_{j-1,k-1})\Big]. \end{aligned} \tag{3}$$

On applying approximations (2) and (3) to initial system of Eqs. (1), its difference analog will have the second order of accuracy

$$\begin{aligned} &\frac{\Delta\hat{q}}{\Delta t} + \delta_\xi\hat{E} + \delta_\eta\hat{F} - \frac{1}{\mathrm{Re}}\left[\delta_\xi\hat{T} + \delta_\eta\hat{S}\right] = \\ &= \frac{\partial\hat{q}}{\partial t} + \frac{\partial\hat{E}}{\partial\xi} + \frac{\partial\hat{F}}{\partial\eta} - \frac{1}{\mathrm{Re}}\left(\frac{\partial\hat{T}}{\partial\xi} + \frac{\partial\hat{S}}{\partial\eta}\right) + O(\Delta t, \Delta\xi^2, \Delta\eta^2). \end{aligned} \tag{4}$$

Most of the results given below were obtained by a difference scheme with improved accuracy. As in [8], to construct the difference scheme we used the operators

$$A_\xi f = \frac{1}{6}(f_{j+1} + 4f_j + f_{j-1}) = \left(1 + \frac{\Delta\xi^2}{6}\frac{\partial^2}{\partial\xi^2}\right) f + 0\,(\Delta\xi^4)\,,$$
$$A_\eta f = \frac{1}{6}(f_{k+1} + 4f_k + f_{k-1}) = \left(1 + \frac{\Delta\eta^2}{6}\frac{\partial^2}{\partial\eta^2}\right) f + 0\,(\Delta\eta^4)\,. \tag{5}$$

Applying the operators A_ξ and A_η to (4), we arrive at the following difference scheme:

$$A_\xi A_\eta \frac{\Delta\hat{\mathrm{q}}}{\Delta t} + A_\eta \delta_\xi \hat{\mathrm{E}} + A_\xi \delta_\eta \hat{\mathrm{F}} - \frac{1}{\mathrm{Re}} A_\xi \delta_\eta \hat{\mathrm{S}} - \frac{1}{\mathrm{Re}} A_\eta \delta_\xi \hat{\mathrm{T}} =$$
$$= \frac{\partial\hat{\mathrm{q}}}{\partial t} + \frac{\partial\hat{\mathrm{E}}}{\partial\xi} + \frac{\partial\hat{\mathrm{F}}}{\partial\eta} - \frac{1}{\mathrm{Re}}\left(\frac{\partial\hat{\mathrm{T}}}{\partial\xi} + \frac{\partial\hat{\mathrm{S}}}{\partial\eta}\right) +$$
$$+ \left(\frac{\Delta\xi^2}{6}\frac{\partial^2}{\partial\xi^2} + \frac{\Delta\eta^2}{6}\frac{\partial^2}{\partial\eta^2}\right)\left\{\frac{\partial\hat{\mathrm{q}}}{\partial t} + \frac{\partial\hat{\mathrm{E}}}{\partial\xi} + \frac{\partial\hat{\mathrm{F}}}{\partial\eta}\right\} + 0\,(\Delta t, \Delta\xi^4, \Delta\eta^4)\,. \tag{6}$$

By virtue of (1), the expression in the braces has the order of $1/\mathrm{Re}$ and in general the error of scheme (6) is $O(\Delta t, \Delta\tau^3, \Delta\eta^4, \Delta\xi^2/\mathrm{Re}, \Delta\eta^2/\mathrm{Re})$ on a nonextended model. Since, as usual, $1/\mathrm{Re} \ll \min(\Delta\xi^2, \Delta\eta^2)$, the scheme will be lower than the fourth order through practically the entire flow region.

Jacobi matrices $\hat{\mathrm{A}} = \partial\hat{\mathrm{E}}/\partial\hat{\mathrm{q}}$, $\hat{\mathrm{B}} = \partial\hat{\mathrm{F}}/\partial\hat{\mathrm{q}}$, which are necessary for linearization of difference scheme (6) in time, have the form

$$\hat{\mathbf{A}}, \hat{\mathbf{B}} = \begin{bmatrix} 0 & k_1 & k_2 & 0 \\ -u\vartheta + k_1\Phi^2 & -(\gamma-2)k_1 u + \vartheta & k_2 u - (\gamma-1)k_1 v & (\gamma-1)k_1 \\ -v\vartheta + k_2\Phi^2 & k_1 v - (\gamma-1)k_2 u & -(\gamma-2)k_2 v + \vartheta & (\gamma-1)k_2 \\ \vartheta\left[\frac{-\gamma e}{\rho} + 2\Phi^2\right] & \left[\frac{\gamma e}{\rho} - \Phi^2\right]k_1 - (\gamma-1)\vartheta u & \left[\frac{\gamma e}{\rho} - \Phi^2\right]k_2 - (\gamma-1)\vartheta u & \gamma\vartheta \end{bmatrix},$$

where $\Phi^2 = 0.5(\gamma-1)(u^2+v^2)$, $\vartheta = k_1 u + k_2 v$.

In linearization of the viscous terms, the mixed derivatives are approximated according to an explicit scheme; therefore, the Jacobi matrix $\hat{\mathbf{M}}$ is written in the following form:

$$\hat{\mathbf{M}} = \frac{1}{J}\begin{bmatrix} 0 & 0 & 0 & 0 \\ m_{21} & \alpha_1 \frac{\partial}{\partial\eta}(\rho^{-1}) & \alpha_2 \frac{\partial}{\partial\eta}(\rho^{-1}) & 0 \\ m_{31} & \alpha_3 \frac{\partial}{\partial\eta}(\rho^{-1}) & \alpha_4 \frac{\partial}{\partial\eta}(\rho^{-1}) & 0 \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix},$$

$$m_{21} = -\alpha_1 \frac{\partial}{\partial\eta}\left(\frac{u}{\rho}\right) - \alpha_2 \frac{\partial}{\partial\eta}\left(\frac{v}{\rho}\right), \quad m_{31} = -\alpha_2 \frac{\partial}{\partial\eta}\left(\frac{u}{\rho}\right) - \alpha_3 \frac{\partial}{\partial\eta}\left(\frac{v}{\rho}\right),$$

$$m_{41} = \alpha_1 \frac{\partial}{\partial\eta}\left[\frac{-e}{\rho^2} + \frac{u^2+v^2}{\rho}\right] - \alpha_1 \frac{\partial}{\partial\eta}\left(\frac{u^2}{\rho}\right) - 2\alpha_2 \frac{\partial}{\partial\eta}\left(\frac{uv}{\rho}\right) - \alpha_3 \frac{\partial}{\partial\eta}\left(\frac{v^2}{\rho}\right),$$

$$m_{42} = -\alpha_4 \frac{\partial}{\partial\eta}\left(\frac{u}{\rho}\right) - m_{21}, \quad m_{43} = -\alpha_4 \frac{\partial}{\partial\eta}\left(\frac{v}{\rho}\right) - m_{31}, \quad m_{44} = \alpha_4 \frac{\partial}{\partial\eta}\left(\frac{1}{\rho}\right),$$

$$\alpha_1 = \mu\left(\frac{4}{3}\eta_x^2 + \eta_y^2\right), \quad \alpha_2 = \frac{\mu}{3}\eta_x\eta_y, \quad \alpha_3 = \mu\left(\eta_x^2 + \frac{4}{3}\eta_y^2\right), \quad \alpha_4 = \frac{\gamma k}{\mathrm{Pr}}(\eta_x^2 + \eta_y^2).$$

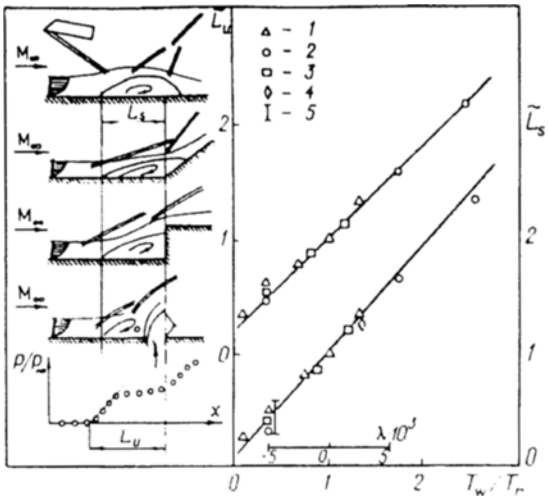

Fig. 1. Flow diagrams. Dependences of the length of the effect against the flow and of the position of the separation point on the characteristics of heat and mass transfer. 1) heat transfer, $\theta = 13^\circ$; 2) same, 10°; 3) mass transfer, $\theta = 13^\circ$; 4) mass transfer [3]; 5) heat transfer [4].

The Jacobi matrix $\hat{\mathbf{N}}$ is written similarly.

Linearizing the flow vectors with respect to the previous time step and employing approximate factorization, we obtain a difference equation which has a block-tridiagonal structure and is solved in fractional steps by vector sweeps:

$$(A_\xi + h\delta_\xi \hat{\mathbf{A}}^n - h\mathrm{Re}^{-1}\delta_\xi \hat{\mathbf{N}}^n)(\Delta\hat{\mathbf{q}}^*)^n =$$

$$= -\Delta t\,(A_\eta\delta_\xi\hat{\mathbf{E}}^n + A_\xi\delta_\eta\hat{\mathbf{F}}^n - \mathrm{Re}^{-1}A_\eta\delta_\xi\hat{\mathbf{T}}^n - \mathrm{Re}^{-1}A_\xi\delta_\eta\hat{\mathbf{S}}^n)\,, \quad (7)$$

$$(A_\eta + h\delta_\eta \hat{\mathbf{B}}^n - h\mathrm{Re}^{-1}\delta_\eta \hat{\mathbf{M}}^n)\,\Delta\hat{\mathbf{q}}^n = \Delta\hat{\mathbf{q}}^{*n}\,,$$

$$\hat{\mathbf{q}}^{n+1} = \hat{\mathbf{q}}^n + \Delta\hat{\mathbf{q}}^n\,,$$

where $h = \Delta t$ or $h = \Delta t/2$ for the first or second orders of accuracy in time.

The derivatives in the boundary conditions were approximated by corresponding explicit and implicit one-sided difference operators.

The considered numerical algorithm is implemented, together with others, by explicit, mixed and implicit numerical methods within the framework of a unique package of application programs for solving the Euler and Navier–Stokes equations. All the algorithms were tested on the problem of the interaction between a shock wave and a laminar boundary layer [9, 14].

A comparison of the above-presented algorithm and the Steger method showed [14] that the introduction of additional operators A_ξ and A_η does not lead to an appreciable increase in computer time but allows one to obtain qualitative solutions on relatively rough grids involving 30–50 nodes in one direction.

4. Discussion of the Results. We consider the effect of four types of actions associated with distributed heat and mass transfer on a separated flow formed as a result of interaction between an oblique shock wave and a turbulent boundary layer on a plate with a porous section. The oncoming flow parameters are $M_\infty = 2.9$, $\mathrm{Re}_{\delta_0} = 10^6$, $\delta_0 = 0.017$ m, the angle of flow behind the oblique shock wave $\theta = 7$, 10, and 13°. The length of the porous section was equal to 2/3 of the length of the computation region. The flow scheme and the computation region are shown in Fig. 1.

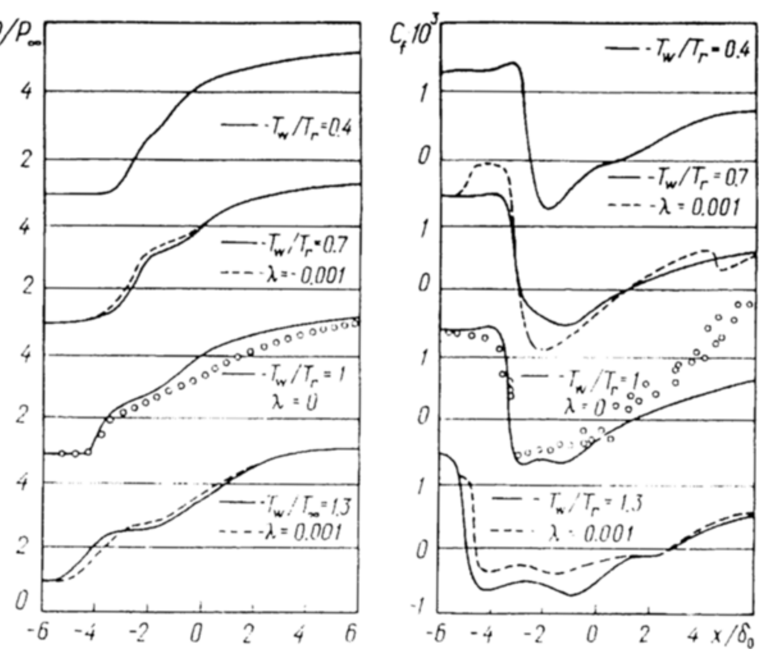

Fig. 2. Distributions of pressure and coefficient of friction in interaction between an oblique shock wave and a turbulent boundary layer. Points, experiment [12].

The averaged Navier–Stokes equations closed by a model of eddy viscosity were used as initial equations to study supersonic turbulent separated flows. In form they do not differ from Eqs. (1). In this case the coefficients of viscosity and thermal conductivity are equal to the sum of the molecular and eddy viscosities.

Several models of eddy viscosity were used to close system of Eqs. (1). Algebraic models of the Cebeci–Smith [12] and Boldwin–Lomax [13] types and the Glushko–Rubezin and Jones–Launder models of the transfer of turbulence characteristics [12] were used.

The calculations were performed on (40×35) and (48×40) node grids. Along the normal to the streamlined surface, the nodes of the physical grid were concentrated according to a power law, so that in the boundary layer there were from 1/2 to 2/3 of the total number of points and allowance for a laminar sublayer in the turbulent flow was provided. Computer time expenditures for the ES-1045 computer were 10–15 h per version. It should be noted that for the given problem only two metric coefficients ξ_x and η_y differ from zero. Since in the considered algorithm all four metric coefficients are present, the computer time expenditures for the numerical experiment are equivalent to the expenditures for the calculation of the development of a supersonic turbulent separation for an arbitrary configuration of the body.

First, flow separation on an impermeable plate with different values of the ratio of surface temperature T_w to recovery temperature T_r within the range from 0.1 to 1.3 was studied [11]. In the second case, the injection parameter, which characterized the mass transfer between oncoming and injected transverse flows $\lambda_\infty = (\rho v)_{inj}/(\rho u)_\infty$, was varied within the limits of from -0.005 to $+0.005$ for conditions of approximate heat insulation of the plate. The third type of effects was characterized by injected gas cooling at a fixed value of λ_∞. The injected gas temperature T_{inj} varied within the range of $(0.1 - 1.0)T_r$ at $T_w/T_r = 1$ on the impermeable portions of the plate. The last to be modeled was separated flow on a free porous surface, i.e., in the absence of pressure and temperature gradients and free gas overflow through a porous section $(\partial p/\partial y = \partial T/\partial y = \partial v/\partial y = 0)$.

Analysis of the results showed a substantial effect of heat and mass transfer conditions on the development of separated supersonic turbulent flow (Figs. 1-5).

Plate cooling leads to reduction of the separation zone, convergence of the separation and reattachment waves, and disappearance of the "plateau" in the pressure distribution. As the plate temperature rises, the boundary layer thickness increases. Thickening of the subsonic region causesfacilitates more intense transfer of disturbances viscous terms; n, number of time layer; p, static pressure in a flow; Pr, Prandtl number; $\hat{\mathbf{q}}$, vector of dependent variables; Re, Reynolds number; t, time; T, temperature; $\hat{\mathbf{T}}$, $\hat{\mathbf{S}}$, vectors of viscous terms; u, v, components of velocity vector in Cartesian coordinates; x, y, Cartesian coordinates; ξ, η, curvilinear coordinates related to surface of streamlined body; ξ_x, ξ_y, η_x, η_y, metric coefficients of the transformation of coordinates; λ, coefficient of bulk viscosity ($\lambda = -2/3\mu$); ρ, density; μ, dynamic coefficient of viscosity; k, total coefficient of thermal conductivity; γ, ratio of specific heat capacities; τ, shear stresses; δ, central difference operator; Δ, ∇, operators of right and left differences; $\Delta\xi$, $\Delta\eta$, steps of uniform coordinate grid toward ξ and η, respectively; δ_0, initial thickness of boundary layer; θ, angle of wedge slope. Indices: j, k, numbers of nodes of computation grid towards ξ and η, respectively; ∞, undisturbed flow; 0, parameters of an adiabatically retarded flow; w, wall.

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